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The Dominating Graph $DG^{bcd}(G)$ of a Graph G

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***Abstract:** The dominating graph $DG^{bcd}(G)$ of a graph G is obtained from G with vertex set $V' = V(G) \cup S$, where $V = V(G)$ and S is the set of all minimal dominating sets of G. Two elements in V' are said to satisfy property 'a' if $u, v \in V'$ and are adjacent in G. Two elements in V' are said to satisfy property 'b' if $u = D_1$, $v = D_2 \in S$ and have a common vertex. Two elements in V' are said to satisfy property 'c' if $u \in V(G)$, $v = D \in S$ such that $u \in D$. Two elements in V' are said to satisfy property 'd' if $u, v \in V(G)$ and there exists $D \in S$ such that $u, v \in D$. A graph having vertex set V' and any two elements in V' are adjacent if they satisfy any one of the property b, c, d is denoted by $DG^{bcd}(G)$. In this paper, we have studied some basic properties of $DG^{bcd}(G)$. We have characterized graphs G for which $DG^{bcd}(G)$ has some specific properties. Also, we have established some extremal properties of $DG^{bcd}(G)$.*

***Keywords:** Dominating set, minimal dominating set, dominating graph. 2010Mathematical Classification Code: 05C12, 05C69.*

I. INTRODUCTION

Graphs discussed in this paper are undirected and simple. For graph theoretic terminology refer to Harary[3], Buckley and Harary[1]. For a graph, let V(G) and E(G) denotes its vertex and edge set respectively. A graph with p vertices and q edges is called a (p, q) graph. The degree of a vertex v in a graph G is the number of edges of G incident with v and it is denoted by deg(v).

The length of any shortest path between any two vertices u and v of a connected graph G is called the distance between u and v and it is denoted by $d_G(u, v)$. The distance between two vertices in different components of a disconnected graph is defined to be ∞. For a connected graph G, The eccentricity e(v) of v is the distance to a vertex farthest from v. Thus, $e(v) = \max\{d(u, v) : u \in V\}$.The radius rad(G) = r(G) is the minimum eccentricity of the vertices, whereas the diameter diam(G) is the maximum eccentricity. If these two are equal in a graph, that graph is called self-centered graph with radius r and is called an r self-centered graph. For any connected graph G, $rad(G) \leq diam(G) \leq 2rad(G)$. v is a central vertex if e(v) = r(G). The center C(G) is the set of all central vertices. For a vertex v, each vertex at a distance e(v) from v is an eccentric vertex of v.

The girth g(G) of the graph G, is the length of the shortest cycle(if any) in G.

A graph G is connected if every two of its vertices are connected, otherwise G is disconnected. The vertex connectivity or simply connectivity $\kappa(G)$ of a graph G is the minimum number of vertices whose removal from G results in a disconnected or trivial graph. The edge connectivity $\lambda(G)$ of a graph G is the minimum number of edges whose removal from G results in a disconnected or trivial graph. A set S of vertices of G is independent if no two vertices in S are adjacent. The independence number $\beta_o(G)$ of G is the maximum cardinality of an independent set.

The concept of distance in graph plays a dominant role in the study of structural properties of graphs in various angles using related concept of eccentricity of vertices in graphs.

The concept of domination in graphs was introduced by Ore[8]. The concepts of domination in graphs originated from the chess games theory and that paved the way to the development of the study of various domination parameters and its relation to various other graph parameters. For details on $\gamma(G)$, refer to [2, 9].

A set $D \subseteq V$ is said to be a dominating set in G, if every vertex in V–D is adjacent to some vertex in D. The cardinality of minimum dominating set is called the domination number and is denoted by $\gamma(G)$. A dominating set D is called a minimal dominating set if no proper subset of D is a dominating set. The upper domination number $\Gamma(G)$ of a graph G is the maximum cardinality of a minimal dominating set of G. Domatic number d(G) of a graph is the largest order of a partition of V(G) into dominating set of G.

For a given graph G, the end edge graph G^+ is the graph obtained from G by adjoining a new edge $u_iu'_i$ at each vertex u_i of G in such a way that this edge has exactly one vertex u_i in common with G.

In [5, 6, 7], Kulli, Janakiram and Niranjan introduced the following concepts in the field of domination theory.

The minimal dominating graph MD(G) of a graph G is the intersection graph defined on the family of all minimal dominating sets of vertices of G[5]. The common minimal dominating graph CD(G) of a graph G is the graph having the same vertex set as G with two vertices adjacent in

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CD(G) if and only if there exists a minimal dominating set in G containing them [6].

The dominating graph D(G) of a graph G = (V, E) is a graph with V(D(G)) = V∪S, where S is the set of all minimal dominating sets of G and with two vertices u, v ∈ V(D(G)) adjacent if u ∈ V and v = D is a minimal dominating set of G containing u[7].

In this paper, we define a new dominating graph $DG^{bcd}(G)$ with property b, c and d. we find some basic properties of $DG^{bcd}(G)$. Also, we characterize graphs G for which $DG^{bcd}(G)$ has some specific properties.

We need the following results to study the dominating graph $DG^{bcd}(G)$ of a graph G.

Theorem: 1.1[5] For any graph G, MD(G) is complete if and only if G contains an isolated vertex.

Theorem: 1.2[4] A graph G is Eulerian if and only if every vertex of G is of even degree.

Theorem: 1.3[4] If for all vertices v of G, deg(v) ≥ p/2 where p ≥ 3, then G is Hamiltonian.

II. THE DOMINATING GRAPH $DG^{BCD}(G)$ OF A GRAPH G

We define a new class of intersection graphs in the field of domination theory as follows.

Definition: 2.1

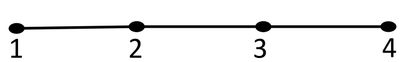
A graph having vertex set V′ = V(G) ∪ S, where S is the set of all minimal dominating sets of G. Two elements in V′ are said to satisfy property 'b' if $u = D_1$, $v = D_2 \in S$ and have a common vertex. Two elements in V′ are said to satisfy property 'c' if u ∈ V(G), v = D ∈ S such that u ∈ D. Two elements in V′ are said to satisfy property 'd' if u, v ∈ V(G) and there exists D ∈ S such that u, v ∈ D. A graph having vertex set V′ and any two elements in V′ are adjacent if they satisfy any one of the property b, c, d is denoted by $DG^{bcd}(G)$.

Here, the elements of V(G) are called as point vertices and the elements of S are known as set vertices.

Clearly, $MD(G) \subseteq DG^{bcd}(G)$, $CD(G) \subseteq DG^{bcd}(G)$, $D(G) \subseteq DG^{bcd}(G)$.

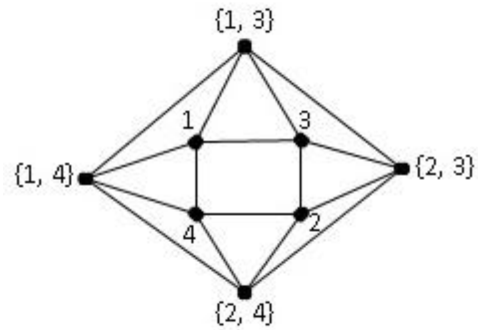
Example:

G:



{1, 3}, {1, 4}, {2, 3}, {2, 4} are minimal dominating sets of G.

$G^{bcd}(G)$:



Remark 2.1: If there is a minimal dominating set having m vertices then $DG^{bcd}(G)$ has K_{m+1} as an induced sub graph. So $K_{\Gamma(G)+1}$ is always an induced sub graph of $DG^{bcd}(G)$, where Γ(G) is the cardinality of the minimal dominating set with maximum number of vertices.

Remark 2.2:

For any graph G, $p+d(G) \leq p' \leq \frac{p(p+1)}{2}$, where d(G) is the domatic number of G and p′ denotes the number of vertices of $DG^{bcd}(G)$.

(i) For any graph G, $2p \leq q' \leq p(p-1)+|S|(|s|-1)/2$, where q′ denoted the number of edges in $DG^{bcd}(G)$.

(ii) $\deg_{DGbcd(G)} v_j = \sum_{j=1}^{p-1} |D_j|$, where D_j denote the minimal dominating sets containing v_j in G.

(iii) $\deg_{DGbcd} D_i \leq |S|-1+|D_i|$, $1 \leq i \leq n$.

Theorem 2.1: If $G = K_p$, then $DG^{bcd}(G)$ is pK_2.

Proof: When $G = K_p$. Each vertex forms a minimal dominating set. Hence we get p such minimal dominating sets. By the definition of $DG^{bcd}(G)$, $DG^{bcd}(G)$ is pK_2.

Theorem 2.2: If $G = K_{1,\,p-1}$, then $DG^{bcd}(G)$ is $K_2 \cup K_p$.

Proof: Let $G = K_{1,\,p-1}$. Let v be the vertex of degree p–1. Since $K_{1,\,p-1}$ has exactly two minimal dominating sets. {v} and all the remaining pendent vertices form minimal dominating sets. By the definition of $DG^{bcd}(G)$, {v} is adjacent to exactly one corresponding vertex of G, and all the remaining vertices are adjacent to another minimal dominating set and each vertex is adjacent to other vertices. Thus, the resulting graph will be $K_2 \cup K_p$. Therefore, $DG^{bcd}(G) = K_2 \cup K_p$.

Corollary 2.1: $\beta_o(DG^{bcd}(G)) = 2$.

We characterize the graphs for which $DG^{bcd}(G)$ is complete.

Theorem 2.3: DG^{bcd} (G) is a complete graph with p+1 vertices if and only if G = $\overline{K_p}$.

Proof: Suppose G = $\overline{K_p}$. The whole vertex set form a minimal dominating set of G. In DG^{bcd}(G), all vertices of G are adjacent to each other and also adjacent to the minimal dominating set. Thus, we get a complete graph with p+1 vertices. Therefore, DG^{bcd} (G) = K_{p+1}.

Conversely, suppose DG^{bcd}(G) is complete. Then each vertex of DG^{bcd} (G) is adjacent. This is possible only the whole vertex set form minimal dominating set. Thus, G = $\overline{K_p}$.

Theorem 2.4: For any graph G, $\overline{G} \subseteq DG^{bcd}$(G).

Proof: If u and v are not adjacent in G. Then {u, v} form an independent set. Thus there exists a minimal dominating set containing both u and v. Thus, it follows that $\overline{G} \subseteq DG^{bcd}$(G).

We obtain a necessary and sufficient condition on a graph G such that DG^{bcd} (G) is connected.

Theorem 2.5: For any graph G with p vertices, $p \geq 2$, DG^{bcd}(G) is connected if and only if $\Delta(G) < p-1$.

Proof: Let $\Delta(G) < p-1$. To prove that DG^{bcd}(G) is connected. Let u, v be any two vertices of G. If u and v are not adjacent in G, then by theorem 2.4, u and v are adjacent in DG^{bcd}(G).

If u and v are adjacent in G and some vertex $w \in V(G)$ distinct from both u and v and is adjacent to neither u nor v. Then there exists two minimal dominating sets D_1 and D_2 such that D_1 and D_2 contain u, w and v, w respectively. Thus u and v are connected in DG^{bcd}(G) by the paths u-w -v, v-D_2-w-u and u-D_1-w-v and D_1, D_2 are adjacent.

Conversely, suppose, DG^{bcd}(G) is connected. To prove that $\Delta(G) < p-1$.

Suppose $\Delta(G) = p-1$ and u is a vertex of degree $p-1$. Then {u} = D is a minimal dominating set of G. Since G has at least two vertices, u is adjacent to all other vertices of G. Thus V−D contains minimal dominating sets. In DG^{bcd}(G), there is no path joining u to any vertex of V−D. This implies that DG^{bcd}(G) is disconnected, a contradiction. Hence, $\Delta(G) < p-1$.

Corollary 2.2: If $\Delta(G) = p-1$, K_2 is a component of DG^{bcd}(G).

We characterize the graphs for which DG^{bcd} (G) is a tree.

Theorem 2.6: DG^{bcd} (G) is a tree if and only if G = K_1.

Proof: Suppose G = K_1, then by the definition of DG^{bcd}(G), DG^{bcd} (G) is K_2.

Conversely, suppose DG^{bcd}(G) is a tree. To prove that G = K_1. Assume G $\neq$ K_1. By theorem 2.5, DG^{bcd}(G) is connected if and only if $\Delta(G) < p-1$. Let $\Delta(G) < p-1$. Then there exists a minimal dominating set D contains u and v, from the definition of DG^{bcd}(G), D, u and v form a triangle, a contradiction. Thus, G = K_1.

Next radius and diameter of DG^{bcd} (G) is found out and classification of graphs such as DG^{bcd}(G) is self-centered with diameter two is discussed.

Theorem 2.7: Let G be a connected graph with $\Delta(G) < p-1$, then in DG^{bcd}(G), $d(u, v) \leq 3$.

Proof: Since $V' = V(G) \cup S$, where S is the set of all minimal dominating sets of G. Let $u, v \in V'$. Consider the following cases.

Case (i)

Suppose $u, v \in V(G)$, $d_G(u, v) > 2$. Then there exists a minimal dominating set D_1 such that D_1 contains u and v. Thus, in DG^{bcd}(G), d(u, v) = 1.

Suppose $d_G(u, v) > 2$, $u \in D_2$ and $v \in D_3$. Then there exists a vertex $z \in V(G)$ is adjacent to both u and v such that $z \in D_2$ and D_3. Thus, it follows that, in DG^{bcd}(G), d(u, v) = d(u, z)+d(z,v) = 2.

Suppose $d_G(u, v) = 1$, $u \in D_4$ and $v \in D_5$. Then there exists a vertex $w \in V(G)$ is neither adjacent to u nor v such that $w \in D_4$ and D_5. This implies that, in DG^{bcd}(G), d(u, v) = d(u, w)+d(w, v) = 2.

Case (ii)

Suppose $u \in V(G)$ and $v \notin V(G)$. Then $v = D'$ is a minimal dominating set of G. If $u \in D'$, then u and v are adjacent in DG^{bcd}(G).

Suppose $u \notin D'$. Consider the following two sub cases.

Sub Case (i)

If $u \in D''$, then there exists a vertex $w \in D'$ such that $w \in D''$. Thus in DG^{bcd}(G), d(u, v) = d(u, w)+d(w, v) = 2 (or) d(u, D′) = d(u, D″)+d(D″, D′) = 2.

Sub Case (ii)

Suppose u lies on only one minimal dominating set D‴. If there exists a minimal dominating set D_6 such that D_6 is adjacent to both D‴ and D′. Thus, it follows that, in DG^{bcd}, d(u, D′) = d(u, D‴)+d(D‴, D_6)+d(D_6, D′) = 3.

Case (iii)

Suppose u, v $\notin$ V(G). Then u = D_7 and v = D_8 are two minimal dominating sets of G. If D_7 and D_8 are not disjoint, then in DG^{bcd}(G), d(u, v) = d(D_7, D_8) = 1. Otherwise, there is a minimal dominating set D_9 such that D_9 is adjacent to both D_7 and D_8. Thus, it follows that, in DG^{bcd}(G), d(u, v) = d(D_7, D_8) = d(D_7, D_9)+(D_9, D_8) = 2.

From the above cases, $d_{DG^{bcd}(G)}(u,v) \leq 3$ is established.

Remark 2.2: If rad(G) > 1, By theorems 2.5 and 2.7, DG^{bcd}(G) is connected and diam(DG^{bcd}(G)) ≤ 3.

Theorem 2.8: Let G be a graph with an isolated vertex. Then rad(DG^{bcd}(G)) = 1 and diam(DG^{bcd}(G)) = 2.

Proof: Suppose G has an isolated vertex. Let u be an isolated vertex and u lies on all the minimal dominating sets of G. Suppose v(≠u) ∈ V(G), we can form a minimal dominating set D such that v ∈ D . Thus all vertices in DG^{bcd}(G) are adjacent to u. Therefore eccentricity of u in DG^{bcd} (G) is one.

Let w, v ∈ V(G). Suppose v ∈ D_1 and w ∈ D_2. Since, in this case every minimal dominating set is adjacent to each other. Thus, in DG^{bcd}(G), d(v, w) = d(v, u)+d(u, w) = 2 and d(D_i, D_j) = 1 for i ≠ j.

Suppose x ∈ V(G) and y $\notin$ V(G). Then y = D_3 is a minimal dominating set of G.

If x $\notin$ D_3. Then in DG^{bcd}(G), d(x, y) = d(x, u)+d(u, y) = 2. Therefore, eccentricity of point vertices is 2 except u and eccentricity of set vertices is 2. Hence, rad(DG^{bcd}(G)) = 1 and diam(DG^{bcd}(G)) = 2.

Theorem 2.9: Let G be a disconnected graph without isolated vertices. Then DG^{bcd}(G) is 2- self-centered .

Proof: Suppose G is a disconnected graph. Then G has at least two components G_1 and G_2. Consider the following cases

Case (i)

Let u ∈ V(G_1), v ∈ V(G_2). Then there exists a minimal dominating set D such that D contains u and v. Thus, in DG^{bcd}(G), d(u, v) = 1.

Suppose x, y ∈ V(G_1) and x ∈ D_1, y ∈ D_2. Then there exists a vertex z ∈ V(G_2) such that z ∈ D_1 and z ∈ D_2 and vice versa. Thus, it follows that, in DG^{bcd}(G), d(x, y) = d(x, z)+d(z, y) = 2.

Case (ii)

Suppose a ∈ V(G_1), b ∈ S. Then b = D_3 is a minimal dominating set of G. If a $\notin$ D_3, then there exists a vertex c from any other components such that c ∈ D_3 and c is adjacent to a, it follows that, in DG^{bcd}(G), d(a, b) = d(a, c)+d(c, b) = 2.

Case (iii)

Suppose u′, v′ ∈ S. Then u′ = D_4 and v′ = D_5. If D_4 and D_5 are not disjoint, then in DG^{bcd}(G), d(D_4, D_5) = 1.

Suppose D_4 and D_5 are disjoint. Then there exists a minimal dominating set D′ such that D′ is adjacent to both D_4 and D_5. Thus, it follows that, in DG^{bcd}(G), d(D_4, D_5) = d(D_4, D′)+d(D′, D_5) = 2.

Hence, eccentricity of point vertices is 2 and eccentricity of set vertices is also 2. Therefore, DG^{bcd}(G) is 2 self-centered.

Theorem 2.10: If D_1, D_2 ⊆ V(G) such that D_1 and D_2 are minimal dominating sets and there exists v ∈ D_1(or D_2) such that there is a minimal dominating set containing v and elements of D_2(or D_1), then DG^{bcd}(G) is 2- self-centered.

Proof: Let G be a connected graph with rad(G) > 1. Suppose D_1 and D_2 are two minimal disjoint dominating sets of G, and there exists v ∈ D_1(or D_2) such that there is a minimal dominating set D_3 containing v and elements of D_2(or D_1). Thus, in DG^{bcd}(G), d(v, D_1(orD_2)) = d(v, D_3)+d(D_3, D_1(orD_2)) = 2. Let v′ be the common element of D_1(orD_2) and D_3. Since v and v′ are adjacent in DG^{bcd}(G). Thus, it follows that, d(v, D_1(orD_2)) = d(v, v′)+d(v′, D_1(or D_2)) = 2. By case (i) and case (iii) of theorem 2.7, eccentricity of all point vertices and set vertices are 2. Hence, DG^{bcd}(G) is 2- self-centered.

Theorem 2.11: If D_1, D_2 ⊆ V(G) such that D_1 and D_2 are minimal disjoint dominating sets and there exists v ∈ D_1(or D_2) such that there is no minimal dominating set containing v and elements of D_2(or D_1), then diam(DG^{bcd}(G)) = 3.

Proof: Let G be a connected graph with rad(G) > 1. Suppose D_1 and D_2 are two minimal disjoint dominating sets and there exists v ∈ D_1(or D_2) such that there is no minimal dominating set containing v and elements of D_2(or D_1). Consider the following cases.

Case (i)

There exists a vertex z ∈ V(G) such that z ∈ D_1(or D_2) and z ∈ D_3 which is adjacent to D_1(or D_2). Thus, it follows that, in DG^{bcd}(G), d(v,D_1(or D_2)) = d(v, z)+d(z, D_3)+d(D_3, D_1(or D_2)) = 3.

There exists vertices x, y ∈ V(G) such that x ∈ D_1(or D_2), y ∈ D_1(or D_2) and {x, y} ∈ D_4 is the minimal dominating set of G. Thus, it follows that, in DG^{bcd}(G), d(v, D_1(or D_2)) = d(v, x)+d(x, y)+d(y, D_1(or D_2)) = 3.

There exists a minimal dominating set D_5 such that D_5 is adjacent to both D_1 and D_2. Thus, it follows that, in $DG^{bcd}(G)$, $d(v, D_1(\text{or } D_2)) = d(v, D_1(\text{or } D_2))+d(D_1(\text{or } D_2), D_3)+d(D_3, D_1(\text{or } D_2)) = 3$.

Case (ii)

Suppose $u, u' \in V(G)$. Then there exists a minimal dominating set D'' such that D''contains u, u'. Then, it follows that, in $DG^{bcd}(G)$, $d(u, u') = 1$.

Suppose $u \in D$ and $v \in D'$. Then there exists a vertex $x' \in V(G)$ such that $x' \in D$ and D'. Thus, if follows that, in $DG^{bcd}(G)$, $d(u, u') = d(u, x')+d(x', u') = 2$.

Case (iii)

Suppose $u, u' \notin V(G)$. Then $u = D_4$ and $u' = D_5$ are two minimal dominating sets of G. If D_4 and D_5 have a common vertex, then in $DG^{bcd}(G)$, $d(D_4, D_5) = 1$. Suppose D_4 and D_5 are disjoint. Then there exists a minimal dominating set D''' such that D''' is adjacent to both D_4 and D_5. Thus, it follows that, in $DG^{bcd}(G)$, $d(D_4, D_5) = d(D_4, D''')+d(D''', D_5) = 2$. So, in all the cases, $diam(DG^{bcd}(G))$ is 3.

Corollary 2.3: If G with $rad(G) > 1$ is a connected graph and there exists a vertex $v \in V(G)$ such that v is in exactly only one minimal dominating set, then $diam(DG^{bcd}(G)) = 3$.

Proof: Proof follows from theorem 2.11.

Theorem 2.12: $DG^{bcd}(G) = K_{p+1}$ if and only if $G = \overline{K_p}$.

Proof: Suppose $G = \overline{K_p}$, then by theorem 2.3 $DG^{bcd}(G)$ is a complete graph with p+1 vertices.

Conversely, suppose $DG^{bcd}(G) = K_{p+1}$. Then $rad(DG^{bcd}(G)) = diam(DG^{bcd}(G)) = 1$. To prove that $G = \overline{K_p}$. On the contrary, assume that $G \neq \overline{K_p}$, G is connected. Then there exists at least two minimal dominating sets and $\Delta(DG^{bcd}(G)) < p'-1$, where p′ is the number of vertices in $DG^{bcd}(G)$, a contradiction. Hence, G must be $\overline{K_p}$.

Theorem 2.13: $rad(DG^{bcd}(G)) = 1$, $diam(DG^{bcd}(G)) = 2$ if and only if G has at least one isolated vertex.

Proof: Suppose G has an isolated vertex. Then by theorem 2.8 $rad(DG^{bcd}(G)) = 1$, $diam(DG^{bcd}(G)) = 2$.

Conversely, suppose $rad(DG^{bcd}(G)) = 1$, $diam(DG^{bcd}(G)) = 2$. To prove that G has at least one isolated vertex. On the contrary, assume that G is disconnected without isolated vertices. Then, by theorem 2.9, $DG^{bcd}(G)$ is 2-self-centered. Hence G has at least one isolated vertex.

Theorem 2.14: $DG^{bcd}(G)$ is 2- self-centered if and only if G is any one of the following:

(i) G is a disconnected graph without isolated vertices.

(ii) G is a connected graph. For $D_1, D_2 \subseteq V(G)$ such that D_1 and D_2 are two minimal disjoint dominating sets of G and there exists $v \in D_1(\text{or } D_2)$ such that there exists a minimal dominating set containing v and elements of $D_2(\text{or } D_1)$.

Proof: Proof follows from theorems 2.9 and 2.10.

Theorem 2.15: $rad(DG^{bcd}(G)) = 2$, $diam(DG^{bc}(G)) = 3$ if and only if $D_1, D_2 \subseteq V(G)$ such that D_1 and D_2 are two minimal disjoint dominating sets of G and there exists $v \in D_1(\text{or } D_2)$ such that there is no minimal dominating set containing v and elements of $D_2(\text{or } D_1)$.

Proof: Proof follows from theorem 2.11.

Following two theorems deals with the domination parameters of $DG^{bcd}(G)$.

Theorem 2.16: $\gamma(DG^{bcd}(G)) = p$ if and only if $G = K_p$.

Proof: Suppose $G = K_p$. By the definition of $DG^{bcd}(G)$, $DG^{bcd}(G)$ is pK_2. Thus, if follows that $\gamma(DG^{bcd}(G)) = p$.

Conversely, suppose $\gamma(DG^{bcd}(G)) = p$. To prove that $G = K_p$. On the contrary, assume that $G \neq K_p$. Then there exists at least two non-adjacent vertices u and v in G. There exists a minimal dominating set D such that D contains u and v. The remaining vertices other than u and v with D form minimal dominating set of $DG^{bcd}(G)$. This implies that $\gamma(DG^{bcd}(G)) \leq p-1$, a contradiction. Hence $G = K_p$.

Theorem 2.17: For any graph G, $1 \leq \gamma(DG^{bcd}(G)) \leq p$. The both upper and lower bounds are sharp.

Proof: Proof follows from theorems 2.16 and 2.17.

Theorem 2.18: If $G = K_p \cup K_1$, for all $p \geq 2$, then K_p^+ and $K_{1,p}$ are edge disjoint sub graphs of $DG^{bcd}(G)$.

Proof: Let $G = K_p \cup K_1$. Let v_i be an isolated vertex in G. Then $deg(v_i)$ is 2p in $DG^{bcd}(G)$. Since MD(G) is an induced sub graph of $DG^{bcd}(G)$. From theorem 1.1, it follows that K_p is a sub graph of $DG^{bcd}(G)$. Clearly, K_p^+ and $K_{1,2p}$ are edge disjoint sub graphs of $DG^{bcd}(G)$.

Next we find out the girth of $DG^{bcd}(G)$.

Theorem 2.19: Let G be a connected graph with $\Delta(G) < p-1$, then girth of $DG^{bcd}(G)$ is three.

Proof: Let G be a connected graph with $\Delta(G) < p-1$. Then $DG^{bcd}(G)$ is connected. If there is a minimal dominating set having m vertices then $DG^{bcd}(G)$ has K_{m+1} as an induced sub graph. Therefore girth of $DG^{bcd}(G)$ is three.

Next we study the connectivity and edge connectivity of $DG^{bcd}(G)$.

Theorem 2.20: For any graph G,

$\kappa(DG^{abc}(G)) \leq \min\{\min\{\deg_{DG^{bcd}(G)}(v_i),\ 1\leq i \leq p,\ \min\{\deg_{DG^{bcd}(G)}|S_j|,\ 1\leq j\leq n\}\}$, where S_j's are the minimal dominating sets of G.

Proof: Let G be a (p, q) graph. We consider the following cases.

Case (i)

Let $v \in V(G)$ having minimum degree among all v_i's. If the degree of v is less than degree of any other vertex in $DG^{bcd}(G)$. Then by deleting those vertices of $DG^{bcd}(G)$, which are adjacent to v, the resulting graph is connected.

Case (ii)

Let u = S, having minimum degree among all vertices of S_j's. If the degree of u is less than degree of any other vertex in $DG^{bcd}(G)$. Then by deleting those vertices of $DG^{bcd}(G)$, which are adjacent to u, the resulting graph is disconnected. Hence,

$\kappa(DG^{abc}(G)) \leq \min\{\min\{\deg_{DG^{bcd}(G)}(v_i),\ 1\leq i\leq p,\ \min\{\deg_{DG^{bcd}(G)}|S_j|,\ 1\leq j\leq n\}\}$.

Theorem 2.21: For any graph G, $\lambda(DG^{abc}(G)) \leq \min\{\min\{\deg_{DG^{bcd}(G)}(v_i),\ 1\leq i\leq p,\ \min\{\deg_{DG^{bcd}(G)}|S_j|,\ 1\leq j\leq n\}\}$, where S_j's are the minimal dominating sets of G.

Proof: Proof is similar to theorem 2.21.

Next two theorems give the traversability properties of the graph $DG^{bcd}(G)$.

Theorem 2.22: Let $\Delta(G) < p-1$ and $\Gamma(G) = 2$. If every vertex is in exactly two minimal dominating sets, then $DG^{bcd}(G)$ is Hamiltonian and also Eulerian.

Proof: Let $\Delta(G) < p-1$ and $\Gamma(G) = 2$. Then $DG^{bcd}(G)$ is connected and $\gamma(G) = \Gamma(G)$. Every minimal dominating set contains two vertices, suppose every vertex is in exactly two minimal dominating sets. Then $v \in V(G)$ and D is a minimal dominating set of G, $\deg_{DG^{bcd}(G)} v = \deg_{DG^{bcd}(G)} D = 4$. Then by theorem 1.2, $DG^{bcd}(G)$ is Eulerian. Since for each $v \in V(G)$, there exists two minimal dominating set containing v. Then $DG^{bcd}(G)$ contains a spanning cycle. Hence it is Hamiltonian.

Theorem 2.23: If G is a (p–2) regular graph, then $DG^{bcd}(G)$ is Eulerian and also Hamiltonian.

Proof: Let G be a (p–2) regular graph. Then any two vertices form the minimal dominating set of G. In $DG^{bcd}(G)$, degree of point vertices is 2(p–1) and degree of set vertices is also 2(p–1). Thus, every vertex of $DG^{bcd}(G)$ has an even degree. Then, it follows that $DG^{bcd}(G)$ is Eulerian. In $DG^{bcd}(G)$, $v \in V'$, $\deg v \geq p'/2$. Thus, it follows that $DG^{bcd}(G)$ is Hamiltonian.

III. CONCLUSION

In this paper, we have defined and studied the new dominating graph $DG^{bcd}(G)$. Eccentricity of vertices of $DG^{bcd}(G)$ is found out. Classification of graphs for which $DG^{bcd}(G)$ is self-centered with diameter two is discussed. Properties such as traversability, connectivity and edge connectivity of $DG^{bcd}(G)$ are established. Domination parameters of $DG^{bcd}(G)$ is also studied.

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