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Boolean Function Graph $B(\overline{G},\overline{K}_q,INC)$ Of A Graph

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***Abstract:** Let G be a simple graph with vertex set V(G) and edge set E(G).The Boolean Function Graph $B(\overline{G},\overline{K}_q,INC)$ of G is a graph with vertex set V(G) ∪ E(G) and two vertices in $B(\overline{G},\overline{K}_q,INC)$ are adjacent if and only if they correspond to two nonadjacent vertices of G or to a vertex and an edge incident to it in G. For simplicity ,This graph is denoted by BF_1(G). In this paper, covering number, independence number, chromatic number and domination number of BF_1(G) are found.*

***Keyword:** covering number, independence number, domination number, Boolean graph.*

I. INTRODUCTION

Graphs discussed in this paper are undirected and simple graphs. For a graph G, let V(G) and E(G) denote its vertex set and edge set respectively. For two vertices u and v in a connected graph G, the distance d(u, v) from u to v is the length of a shortest u-v path in G. A covering of a graph G = (V, X) is a subset K of V such that every line of G is incident with a vertex in K. A covering K is called a minimum covering if G has no covering K' with $|K'| < |K|$. The number of vertices in a minimum covering of G is called the covering number of G and is denoted by α_0(G) or α_0 . A line covering of G is a subset of L of X such that every vertex is incident with a line of L. The number of lines in a minimum line covering of G is called the line covering number of G and is denoted by α_1(G) or α_1 . A subset S of V is called an independent set of G if no two vertices of S are adjacent in G. An independent set S is said to be maximum if G has no independent set S' with |S'| > |S|. The number of vertices in a maximum independent set is called the independence number of G and is denoted by $b_0(G)$ or b_0 . Analogously, an independent set of edges of G has no two of its edges adjacent and the maximum cardinality of such a set is the line independent number of $b_1(G)$or b_1.

The concept of domination in graphs was introduced by Ore [6]. A setD ⊆ V (G) is said to be a dominating set of G, if every vertex in V(G)–Dis adjacent to some vertex in D. D is said to be a minimal dominating set if D – {u} is not a dominating set, for any u ∈ D. The domination number γ(G) of G is the minimum cardinality of a dominating set. We call a set of vertices a γ − set, if it is a dominating set with cardinality γ(G). Whitney [12] introduced the concept of the line graph L(G) of a given graph G in 1932. The first characterization of line graphs is due to Krausz. The Middle graph M(G) of a graph G was introduced by Hamada and Yoshimura[5]. Chikkodimath and Sampathkumar [3] also studied it independently and they called it the semi-total graph T(G) of a graph G. Characterizations were presented for middle graphs of any graph, trees and complete graphs in [1]. The concept of total graphs was introduced by Behzad [2] in 1966. Janakiraman et al., introduced the concepts of Boolean and Boolean function graphs [7 - 10]. The Boolean function graph $B(\overline{G},\overline{K}_q,INC)$ of G is a graph with vertex set V (G) ∪ E(G) and two vertices in $B(\overline{G},\overline{K}_q,INC)$ are adjacent if and only if they correspond to two nonadjacent vertices of G or to a vertex and an edge incident to it in G. For brevity, this graph is denoted by BF_1(G). In this paper, covering number, independence number, chromatic number and domination number are found. Here, G is a graph with p vertices and q edges. The definitions and details not furnished in this paper may be found in [4].

II. PRIOR RESULT

Observation

2.1. V (BF_1(G)) =V (G) ∪ V (L(G)), where L(G) is the line graph of G.

2.2. BF_1(G) is always connected.

2.3. $\overline{G}$and $\overline{K}_q$ are induced subgraphs of BF_1(G).

2.4. The number of vertices in BF_1(G) is p+q and the number of edges in BF_1(G) is $\frac{p(p-1)}{2}$ + q.

2.5. The degree of u ∈V(G) in BF_1(G) is p – 1 and degree of e ∈ E(G) in BF_1(G) is 2

2.6. If p=3, thenBF_1(G)isC_3,C_4orC_5.

2.7. If p = 4, then BF_1(G) is bi-regular.

III. MAIN RESULTS

In the following theorems, bounds of covering number of $BF_1(G)$ are found.

Theorem 3.1: If G has isolated vertices, then $\alpha_0(BF_1(G)) \leq p-1$.

Proof: Let $v_1, v_2, v_3, \ldots, v_n$, $(n \geq 1)$ be isolated vertices in G.

Let $G' = \langle V(G) - \{v_1, v_2, v_3, \ldots v_n\} \rangle$. Then, $\alpha_0(BF_1(G')) \leq p-n$. Since the subgraph of $BF_1(G)$ induced by all the isolated vertices is complete, $\alpha_0(\langle \{v_1, v_2, v_3, \ldots, v_n\} \rangle) = n-1$.

Therefore, $\alpha_0(BF_1(G)) \leq (p-n)+(n-1) = p-1$.

Theorem 3.2: Let G be a disconnected graph having no isolated vertices and contain K_2 as one of its components. Then $\alpha_0(BF_1(G)) \leq p-1$.

Proof: Let $V(K_2) = \{u,v\}$ and $e = (u,v) \in E(K_2)$. Then $(V(G)-\{u,v\}) \cup \{e\}$ is a point cover of $BF_1(G)$ and hence $\alpha_0(BF_1(G)) \leq p-1$.

Remark 3.1: If $G \cong K_2$, then $BF_1(G) \cong P_3$ and $\alpha_0(BF_1(G)) = 1$.

Theorem 3.3: Let G be a graph such that $\delta(G) \geq 1$. Then $\alpha_0(BF_1(G)) = \gamma(BF_1(G))$ if and only if V(G) is a minimum dominating set of $BF_1(G)$.

Proof: Since $\delta(G) \geq 1, \delta(BF_1(G)) \geq 1$. Therefore, $\gamma(BF_1(G)) \leq \alpha_0(BF_1(G))$. Let D= V(G) be a minimum dominating set of $BF_1(G)$. Since $\overline{G}$ is an induced subgraph of $BF_1(G)$, D covers all the edges $\overline{G}$ and the edges in $BF_1(G)$ of the form (v,e) where $e \in E(G)$ is incident with v in G. Therefore, D covers all the edges in $BF_1(G)$ and $V(BF_1(G))-D$ is a maximum independent set in $BF_1(G)$. Hence, D is a covering for $BF_1(G)$.

Therefore, $\alpha_0(BF_1(G)) \leq \gamma(BF_1(G))$. Conversely, assume

$\alpha_0(BF_1(G)) = \gamma(BF_1(G))$. Therefore any minimum covering of $BF_1(G)$ is a minimum dominating set of $BF_1(G)$. Let D = V(G). Then $V(BF_1(G)) - D$ is a maximum independent set of $BF_1(G)$ and hence D is a minimum covering of $BF_1(G)$. Therefore, D is a minimum dominating set of $BF_1(G)$.

In the following theorems, bounds of independence number of $BF_1(G)$ is found.

Theorem 3.4: For any (p,q) graph G, $\beta_0(BF_1(G)) \geq \max[q, \omega(G)+m]$, where $\omega(G)$ is the clique number of G and m is the number of edges in G not incident with the vertices in the maximal clique

Proof: The subgraph of $BF_1(G)$ induced by all the vertices in L(G) is totally disconnected. Therefore, $\beta_0(BF_1(G)) \geq q$. Let D be a maximal clique in G and let D' be the set of all vertices in $BF_1(G)$ corresponding to the edges in G not incident with the vertices in the maximal clique. Then $D \cup D'$ is an independent set in $BF_1(G)$,

$\beta_0(BF_1(G)) \geq |D \cup D'| = \omega(G) + m$. Therefore,

$\beta_0(BF_1(G)) \geq \max[q, \omega(G) + m]$.

Observation 3.1: Let G be a graph, such that $\delta(G) \geq 1$. Then $\beta_0(BF_1(G)) = \gamma(BF_1(G))$ if and only if the set of vertices of $BF_1(G)$ corresponding to the edges of G is a minimum dominating set of $BF_1(G)$.

In the following theorems, bounds of line covering number of $BF_1(G)$ is found.

Theorem 3.5: Let G be a graph, such that $\delta(G) \geq 1$ and $\delta(\overline{G}) \geq 1$. Then $\alpha_1(BF_1(G) \geq \min(\max(p, q), \beta_1(\overline{G}) + q)$

Proof: Case (i): $q \geq p$.

Let S be the set of all the edges of the form (v,e) in $BF_1(G)$, where

$e \in E(G)$ is incident with $v \in V(G)$. For each edge e in G, there is an edge of the form (v,e) in $BF_1(G)$. Then |S|= q and S is a line cover for G and hence $\alpha_1(BF_1(G)) \leq q$.

Case (ii) $q < p$,

The set of all edges of the form (v_i, e_{ij}); i= 1,2,3,...,p is a line cover for $BF_1(G)$. Let D be a matching in $\overline{G}$ such that $|D| = \beta_1(\overline{G})$ and Let T be the set of vertices in $\overline{G}$ not incident with the edges in D. Then $D \cup \{(v_i, e_{jk})\}$ is a line cover for $BF_1(G)$ and

$\alpha_1(BF_1(G) \leq \beta_1(\overline{G}) + q$. Therefore, $\alpha_1(BF_1(G) \leq \min[p, \beta_1(\overline{G}) + q]$. Hence, $\alpha_1(BF_1(G)) \geq \min(\max(p,q), \beta_1(\overline{G}) + q)$.

Remark 3.2: Let G be any graph such that both G and $\overline{G}$ have no isolated vertices and let D be a set containing maximum number of independent edges in G.If S is the set of vertices in $\overline{G}$ not incident with the edges in D, then $\beta_1(BF_1(G) \geq \max\{\min(p, q), \beta_1(\overline{G}) + |S|\}$.

In the following theorems, Chromatic number of $BF_1(G)$ is found.

Theorem 3.6: If $\chi(\overline{G}) = 2$ and if each pair of nonadjacent vertices in $\overline{G}$receives the same colour, then $\chi(BF_1(G)) = 2$.

Proof: Assume $\chi(\overline{G}) = 2$. Let u and v be two nonadjacent vertices in $\overline{G}$ and u and v receive the same colour, Then, $e = (u, v) \in E(G)$ and $e\in V(BF_1(G))$. By assumption, $u,v \in V(BF_1(G))$ hence in $BF_1(G)$ receive the same colour. Colour the vertex e in $BF_1(G)$ with the second colour. Therefore, $BF_1(G)$ is 2-colourable.

Hence $\chi(BF_1(G)) = 2$.

Theorem 3.7: If $\chi(\overline{G}) = 2$ and if there exists atleast one pair of nonadjacent vertices in $\overline{G}$ receiving different Colours,then $\chi(BF_1(G)) = 3$.

Proof: Let $\chi(\overline{G}) = 2$ and $(u, v) \notin E(\overline{G})$ such that u and v receive different colours in $\overline{G}$. Therefore $e = (u, v) \in E(G)$.Then the vertex

$e \in V(BF_1(G))$ is coloured with the third colour and hence $\chi(BF_1(G)) = 3$.

Theorem 3.8: If $\chi(\overline{G}) = k$, $(k \geq 3)$, then $\chi(BF_1(G)) = k$.

Proof: Let $\chi(\overline{G}) = k$, $(k \geq 3)$ and $e_{ij} = (v_i,v_j) \in E(G)$.Then

$e_{ij}\in V(BF_1(G))$.Since any two vertices of L(G) are nonadjacent in $BF_1(G)$, colour the vertex $e_{ij}$by a colour other than the colours used for $v_i$and v_j in $\overline{G}$. Therefore, $BF_1(G)$ is k-colourable and $\chi(BF_1(G)) = k$.

Theorem 3.9: Edge chromatic number of $\chi'(BF_1(G))$ of $BF_1(G)$ is (p – 1) or p.

Proof: When $p \geq 3$, maximum degree of $BF_1(G) = \Delta(BF_1(G)) = p - 1$. Therefore, $\chi'(BF_1(G)) = \Delta(BF_1(G))$ or $\Delta(BF_1(G))+1=(p-1)$ or p.

In the following theorems, bounds of domination number of $BF_1(G)$ are found.

Observation 3.2: For any graph G,$\gamma(BF_1(G))=1$if and only if G is isomorphic to K_2ornK_1,$n\geq1$.

Theorem 3.10: For any graph G,$\gamma(BF_1(G)) = 2$ if and only if either (a) There exits a point cover D of G with $|D|=2$ and for any vertex $v\in V - D$, $N(v)\cap D$ is properly contained in D (or)

(b) G is one of the graphs $2K_2$, P_3, and $K_2\cup mK_1$, $m \geq 1$.

Proof: Assume $\gamma(BF_1(G)) = 2$

Let D be a dominating set of $BF_1(G)$ with $| D |= 2$

Case (1): $D = \{v_1, v_2\}$, $v_1, v_2\in V(G)$. Since D dominates all the vertices of L(G) in $BF_1(G)$, each edge in G is incident with a vertex in D. That is, D is a point cover for G. If $N(v)\cap D = D$, for some $V(G)-D$, then v is adjacent to v_1,$v_2$in D. But $\overline{G}$is an induced subgraph of $BF_1(G)$. Therefore, $v \in V(BF_1(G)) - D$ is not adjacent to any of the vertices in D. Hence, $N(v) \cap D$ is properly contained in D.

Case(2): D = {v,e}, where $v \in V(G)$, $e \in V(L(G))$. If there exits an edge other than e in G, then the vertex in $BF_1(G)$ corresponding to e is neither adjacent to v nor e in $BF_1(G)$. Hence, $G \cong K_2\cup mK_1$, $m \geq 1$. If m = 0, then $\gamma(BF_1(G)) =1$.

Case (3): $D= \{e_1, e_2\}$, where $e_1, e_2\in V(L(G))$. Since no two vertices in L(G) are adjacent in $BF_1(G)$, G contains no edges other than $e_1$and e_2. Also, the vertices of G which are incident with $e_1$and $e_2$alone adjacent to $e_1$and $e_2$in $BF_1(G)$. Therefore, $G \cong 2K_2$or P_3. Conversely, if (a) holds (or) (b) then $\gamma(BF_1(G)) = 2$.

In similar lines, the following theorem can be proved.

Theorem 3.11: For any graph G, any set $D \subseteq V(G)$ is a dominating set of $BF_1(G)$ if and only if

(i) D is a point cover of G

(ii) (ii) $N(v) \cap D \subset D$, for all $v \in V(G) - D$.

Theorem 3.12; Let G be any graph having no isolated vertices. For each minimum point cover D in G, if there exists a vertex $v\in V(G) - D$ such that the subgraph of G induced by $D \cup \{v\}$ contains a star with $|D| + 1$ vertices (with v as the central vertex) as a subgraph, then $\gamma(BF_1(G)) \leq \alpha_0(G) +1$.

Proof: Let D be a minimum point cover of G.Then $D \subseteq V(BF_1(G))$ dominates all the vertices of L(G)in $BF_1(G)$. Let there exists a vertex $v \in V(G) - D$ such that the subgraph of G induced by $D \cup \{v\}$ contains a star with $| D | + 1$ vertices and v as the central vertex. Then $v \in V(BF_1(G)) - D$ is not adjacent to any of the vertices in D. But the subgraph of $BF_1(G)$ induced by $V(G) - D$ is complete. Therefore, to dominate the vertices of G in $V(BF_1(G)) - D$, a vertex of G is required. Therefore, $\gamma(BF_1(G)) \leq \alpha_0(G) + 1$.

Remark 3.3: For any graph G, $\gamma(BF_1(G)) \leq \alpha_0(G)$ if and only if there exists a minimum point cover D of G such that D is a dominating set of $\overline{G}$. That is $\alpha_0(G) = \gamma(\overline{G}) \leq \chi(G)$.

Example 3.1: 1. $\gamma(BF_1(C_n)) = \alpha_0(C_n)$, $n \geq 5$.

2. $\gamma(BF_1(C_4)) = \alpha_0(C_4) + 1 = 3$.

3. $\gamma(BF_1(P_n)) = \alpha_0(P_n)$, $n \geq 6$.

4. $\gamma(BF_1(P_5)) = \alpha_0(P_5) + 1 = 3$.

5. $\gamma(BF_1(P_4)) = \alpha_0(P_4) = 2$.

Theorem:3.13: For any graph G, V(L(G)) is a dominating set of $BF_1(G)$ if and only if G contains no isolated vertices.

Proof: Let D= V(L(G)) be a dominating set of $BF_1(G)$. Let v be an isolated vertex in G. Then $v \in V(BF_1(G))$ is not adjacent to any of the vertices in D. Therefore, G contains no isolated vertices. Conversely, if G contains no isolated vertices, then each vertex in $V(BF_1(G))-D$ is adjacent to atleast one vertex in D.

Remark:3.4: Any proper subset of V(L(G)) is not a dominating set of $BF_1(G)$, since the subgraph of $BF_1(G)$ induced by vertices of L(G) is totally disconnected.

Theorem:3.14: For any graph G, $\gamma(BF_1(G)) \leq \gamma(\overline{G}) + \alpha_0(\langle V(G)-D \rangle)$, Where D is a minimum dominating set of $\overline{G}$

Proof: Let D be a dominating set of $\overline{G}$ with $|D| = \gamma(\overline{G})$. Then $D \subseteq V(BF_1(G))$ dominates all the vertices of G in $BF_1(G)-D$ and the vertices of L(G) corresponding to the edges in G, incident with the vertices in D. A point cover of $\langle V(G)-D \rangle$ dominates the remaining vertices of L(G) in $BF_1(G)$. Hence, $\gamma(BF_1(G)) \leq \gamma(\overline{G}) + \alpha_0(\langle V(G)-D \rangle)$.

Theorem:3.15: For any graph G, $\gamma(BF_1(G)) \leq \alpha_1(G) + \alpha_0(\langle E(G)-D \rangle)$, Where D is a minimum line cover of G.

Proof: Let $D \subseteq E(G)$ be a minimum line cover of G . Therefore, $|D| = \alpha_1(G)$. Let D' be a subset of $V(BF_1(G))$ corresponding to the edges in D. Then $D' \subseteq V(BF_1(G))$ dominates all the vertices of G in $V((BF_1(G))-D')$. The vertices of L(G) in $V(BF_1(G))-D'$ are dominated by vertices of D' . Hence, $\gamma(BF_1(G)) \leq \alpha_1(G) + \alpha_0(\langle E(G)-D \rangle)$.

Theorem:3.16: Let G be a graph with diam(G)=2. If there exists a vertex $v \in V(G)$ such that $\langle N_2(v) \rangle$ is totally disconnected, then $\gamma(BF_1(G)) \leq \Delta(G) + 1$.

Proof: Let $v \in V(G)$ be a such that $\langle N_2(v) \rangle$ is totally disconnected. Let $D = N(v) \cup \{v\}$. Then $D \subseteq V(BF_1(G)) \cap V(G)$. Since $\langle N_2(v) \rangle$ is totally disconnected , all the edges of G incident with the vertices in D. Therefore, vertices of $(BF_1(G)-D)$ corresponding to the edges of G are adjacent to atleast one vertex in D. Also, the vertices of $V(BF_1(G)-D)$ corresponding to vertices in $V(G)-D$ are adjacent to v. Therefore $\gamma(BF_1(G)) \leq |S'| = |N(v)| + 1 \leq \Delta(G) + 1$.

Remark:3.4: Let G be graph with diam(G) = 2. If there exists a vertex $v \in V(G)$ such that $\langle N(v) \rangle \cong K_2 \cup nK_1 (n \geq 0)$ and $\langle N_2(v) \rangle$ is totally disconnected, then $\gamma(BF_1(G)) = 3$, Since the set {v,u,w} is a minimum dominating set of $BF_1(G)$, Where $V(K_2)$={u,w}

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