

Load Flow Analysis for Unbalanced Radial Distribution Systems

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Abstract: A New methodology is developed for handling load flow problem in Radial distribution system. The distinctive topological characteristics of radial distribution networks have been fully exploited to make the direct solution possible. The proposed algorithm uses effective topological and primitive impedance which can be applicable for both balanced and unbalanced radial distribution systems. Due to the direct solution techniques of the proposed method, the time-consuming LU decomposition or Bus admittance matrix required in the traditional load flow methods are no longer necessary. The direct distribution load flow (DLF) algorithm is very much robust and needs very less memory for any size of the distribution systems. The proposed method is tested on various single phase standard IEEE test systems and also on unbalanced 3-phase distribution systems by using MATLAB. Test results demonstrate the validity of the approached method. The approach shows excessive prospective to be used in distribution automation applications.

Keywords: Load Flow, Power Flow, Radial Distribution, DLF, Unbalanced Radial Distribution.

I. INTRODUCTION

Distribution Automation gains importance day-by-day both theoretically and practical implementation. Distribution power flow solution techniques plays a vital Role in automation applications.it also used in various applications like optimization, VAR planning and switching etc. Due to radial structure and high resistance in distribution network, various traditional methods like Gauss-siedel method, Newton – Raphson(N-R)[1] method and Fast decoupled power flow (FDPF) solution techniques makes uneconomical. Consequently many power flow algorithms especially suited for distribution system have emerged day-by day.

The Deterministic Load Flow Algorithm [2] is computationally efficient provides a good convergence property. This load flow algorithm is independent of load model, nos. of laterals and sub-laterals, nos. of buses and R/X ratio of conductors. It can be used for online as well as offline applications Shrimoharhamadi *et al* [3] described a power flow method for solving weakly meshed distribution and transmission networks, using a multi-port compensation technique and basic formulations of Kirchhoff's laws. Extensive study of the performance of this compensation –based power flow scheme that is significantly more efficient than the Newton–Raphson power flow technique when used for solving radial and weakly meshed distribution and transmission networks, Juan A. Martinez reviews different load flow techniques[4] to determine voltage magnitude and phase angle at each bus. Modelling of the distributed network by different load flows is

considered and results are compared with EMT (electromagnetic transients) tool. Luo and semlyen[5] proposed a method for calculating the load flow solution weakly meshed distribution and transmission networks. It uses active and reactive powers as flow variables rather than complex currents. Kersting[6] developed a method for solving the load flow problem in radial distribution networks based on ladder network theory in the iterative routine. This method displays good convergences characteristics for radial systems. Ladder network methods are quite suitable for one sending end radial networks with high R/X ratio. Goswami and basu [7] introduced a power flow solution technique for radial distribution networks. The special topology of distribution networks has been fully exploited to make direct solution possible. . However, the limitation of this method is that no node in the network is the junction of more than three branches, i.e., one incoming and two outgoing branches. Cheng and Shrimoharhamadi [8] presented a three phase power flow solution method for real time analysis of primary distribution systems .this method is a direct extension of the compensation based power flow method for weakly meshed distribution systems[3] from single to three phase networks. M.H.Haque [9] proposed a method of solving the load flow problem of a distribution system. The method can be applied for both radial and meshed networks. A mesh network is converted to a radial network by breaking the loops by adding some dummy buses. The power injections at the loop break points in the equivalent radial network are computed through a reduced order node impedance matrix. In order to solve the unbalanced radial distribution system, K.Krushna Murthy *etc.* develops a backward and forward sweep Algorithm(BFSA)[11].This proposed algorithm utilises simple and flexible numbering scheme and takes full advantage of radial structure of distribution system. In this technique, radial network is restructured by giving the buses and sections using flexible numbering scheme. Ranjan and Das [12] presented a simple and efficient algorithm to solve radial distribution networks. It solves the simple algebraic recursive expression of voltage magnitude and all the data are stored in vector form. D. P. Sharma *etc.*[13] uses the properties of tree to describe a new efficient load flow algorithm. Due to tree type structure of Radial Distribution Network (RDN), it can be modelled as graph and finally adjacency list is used for the efficient representation of RDN in computer memory. Tanveer Husain *etc.*, [14] deals with iterative techniques due to there are no known analytical method to solve the problem. This resulted nonlinear set of

equations or called power flow equations are generated. An efficient and simple load flow method is proposed for analysis of the radial and weakly meshed network based on network topology and basic circuit laws (KCL and KVL)..

The proposed load flow algorithm[10] requires formation of bus-injection to branch current (BIBC) matrix with 1's & 0's as elements and branch-current to bus voltage(BCBV) matrix with primitive impedances as elements & distribution load flow (DLF) matrix. DLF matrix is obtained as product of (BCBV) and (BIBC) matrices. These three matrices require large memory space. In this paper, the main objective function is to form the DLF matrix without the use of BCBV and BIBC matrices. This can be achieved by using branch path matrix (K).

II. BUILDING OF VARIOUS MATRICES IN PROPOSED LOAD FLOW ALGORITHM

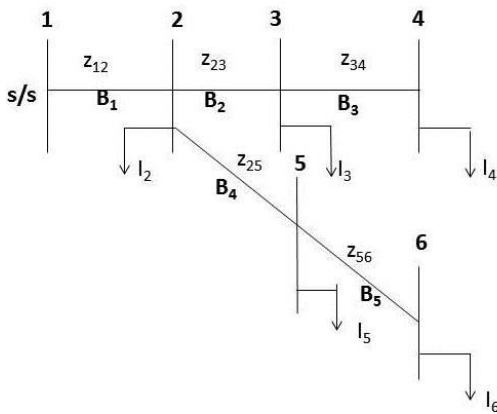


Fig. 1. Simple 6 Bus Radial System Image

The DLF impedance matrix can be obtained by multiplication of BIBC and BCBV matrices. Consider a simple radial distribution network as shown in the fig:1 for formation of proposed load flow algorithm.

A. Structure Of Bus Injection To Branch Current (BIBC) Matrix:

For distribution systems, the complex power at bus i is given by eq (1) and its corresponding equivalent current is given by eq (2).

$$S_i = (P_i + jQ_i) \quad i=1,2,\dots,n \quad (1)$$

$$I_i = (S_i / V_i)^* \quad (2)$$

The current injections at corresponding node for above sample system are as shown in the Fig.1 i.e., I_2, I_3 ..etc.

For the distribution system, shown in Fig.1, apply Kirchhoff's law current law (KCL), the branch currents which is expressed by variable C in terms of equivalent current injections as

$$C_1 = I_2 + I_3 + I_4 + I_5 + I_6 \quad (3)$$

$$C_2 = I_3 + I_4 \quad (4)$$

$$C_3 = I_4 \quad (5)$$

$$C_4 = I_5 + I_6 \quad (6)$$

$$C_5 = I_6 \quad (7)$$

In matrix form it can be represented by

$$\begin{bmatrix} C_1 \\ C_2 \\ C_3 \\ C_4 \\ C_5 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} I_2 \\ I_3 \\ I_4 \\ I_5 \\ I_6 \end{bmatrix}$$

The above branch current equations can be rearranged as below

$$[C] = [BIBC][I] \quad (8)$$

B. Structure Of Bus Injection To Branch Voltage (BIBV) Matrix:

The relationship between the branch currents and bus voltages are given as

$$V_2 = V_1 - C_1 z_{12} \quad (9)$$

$$V_3 = V_2 - C_2 z_{23} \quad (10)$$

$$V_4 = V_3 - C_3 z_{34} \quad (11)$$

$$V_5 = V_2 - C_4 z_{25} \quad (12)$$

$$V_6 = V_5 - C_5 z_{56} \quad (13)$$

On Substitution of (9) & (10) in (11), the voltage at bus 4 is given by

$$V_4 = V_1 - C_1 z_{12} - C_2 z_{23} - C_3 z_{34} \quad (14)$$

Similarly, the other bus voltages can be rewritten as

$$V_3 = V_1 - C_1 z_{12} - C_2 z_{23} \quad (15)$$

$$V_5 = V_1 - C_1 z_{12} - C_4 z_{25} \quad (16)$$

$$V_6 = V_1 - C_1 z_{12} - C_4 z_{25} - C_5 z_{56} \quad (17)$$

Equations (9), (14), (15), (16), (17) can be rearranged as below

$$\begin{bmatrix} V_1 \\ V_1 \\ V_1 \\ V_1 \\ V_1 \end{bmatrix} - \begin{bmatrix} V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \end{bmatrix} = \begin{bmatrix} z_{12} & 0 & 0 & 0 & 0 \\ z_{12} & z_{23} & 0 & 0 & 0 \\ z_{12} & z_{23} & z_{34} & 0 & 0 \\ z_{12} & 0 & 0 & z_{25} & 0 \\ z_{12} & 0 & 0 & z_{25} & z_{56} \end{bmatrix} \begin{bmatrix} C_1 \\ C_2 \\ C_3 \\ C_4 \\ C_5 \end{bmatrix}$$

$$[\Delta v] = [BCBV][C] \tag{18}$$

Now, substitute (8) in (18) and the resulting equation is expressed as

$$[\Delta v] = [DLF][I] \tag{19}$$

Where, $[DLF]$ represents distribution load flow matrix given as

$$\begin{bmatrix} z_{12} & z_{12} & z_{12} & z_{12} & z_{12} \\ z_{12} & z_{12} + z_{23} & z_{12} + z_{23} & z_{12} & z_{12} \\ z_{12} & z_{12} + z_{23} & z_{12} + z_{23} + z_{34} & z_{12} & z_{12} \\ z_{12} & z_{12} & z_{12} & z_{12} + z_{25} & z_{12} + z_{25} \\ z_{12} & z_{12} & z_{12} & z_{12} + z_{25} & z_{12} + z_{25} + z_{56} \end{bmatrix}$$

From the above DLF matrix, the following useful observations are used in developing the proposed topological and primitive based distribution load flow method

1. All elements of DLF matrix of $(n-1) \times (n-1)$ size are complex non-zero and symmetric.
2. Diagonal elements are given by the sum of the primitive impedances of all those lines in the path connecting the substation bus and any selected bus.
3. Each bus-p of the network can have one unique path from substation bus.
4. Off-diagonal p-q elements are given by the sum of the primitive impedances of those lines which appear common to the paths of p and q buses from substation bus.

These observations are effectively used in proposing the algorithm with the help of branch path (k) matrix that exploits the topological structure of the network

III. PROPOSED SOLUTION TECHNIQUE

The proposed method directly determines the Radial distribution load flow solution by simply using primitive impedance of lines and Branch path (K) matrix. There by need of formation of BIBC and BCBV matrix can be avoided. This new algorithm determines the elements of the DLF matrix by comparing rows and columns of the branch path (K) matrix. Thus $[\Delta v]$ elements of the equation [19] can be determined easily.

The proposed approach offers very significant saving in computational burden as it avoids the formation of BIBC and BCBV matrix with exact results at the end.it also requires less iteration for convergence criteria when compared with other solution techniques.

A Typical 6-bus radial distribution network as shown in fig:1 is considered in order to explain the DLF matrix elements formation by using Branch path (K)matrix .

Formation Of DLF Matrix Elements Using Branch Path (K) Matrix:

The K -matrix can be formed from reduced incidence matrix (A)

$$K = \text{transpose (inverse (A))} \tag{20}$$

The K -matrix for stated 6-Bus radial distribution network is given by

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 1 \end{bmatrix}$$

The k matrix is a combination of 0's and 1's.The 1's representation in a row of a Branch path matrix gives information about connecting path between node-1 and any selected node. Thus, diagonal elements of DLF matrix can be formed.

For example, the fourth row in K matrix has 1's in first column and fourth column therefore diagonal element in fourth row is summation of z_{12} and z_{25} i.e.,

$$Z_{44} = z_{12} + z_{25}$$

Similarly, when rows of the K matrix are compared, if there exists a common 1's in respective columns then summation of those primitive impedance will gives respective off diagonal elements of the DLF matrix.

For example, Z_{23} of DLF matrix can be formed by comparing second row and third row which gives summation of z_{12} and z_{23} .i.e.,

$$Z_{23} = z_{12} + z_{23}$$

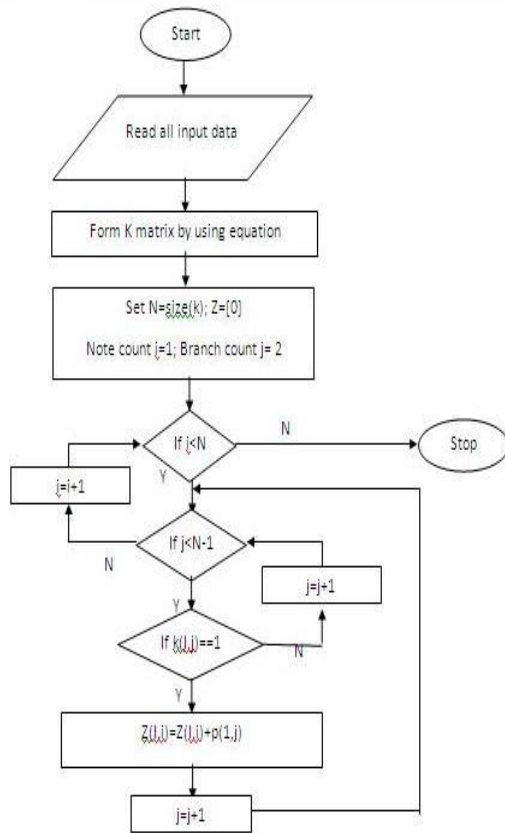


Fig. 2. Flow Chart For Formation Of DLF Matrix

IV. DISTRIBUTION LOAD FLOW ALGORITHM FOR PROPOSED METHOD

- 1 Read the distribution system data and Initialize the Bus Voltages to $1+j0$ p.u.
- 2 Form the Branch path matrix using equation (20).
- 3 Add all the primitive impedances of all those lines by direct comparison of 1's in K matrix as shown in fig:2.
- 4 Calculate the Power Injections and Current Injections $I[i]$ at all the buses
- 5 Assign $I[i]^{old} = I[i]$ for all the buses.
- 6 As discussed in fig:2 calculate the Δv elements of the equation $[\Delta v] = [DLF][I]$
- 7 Update the bus voltages at all the buses.
- 8 Calculate the current Injections $I[i]$ with the updated bus voltages.
- 9 If $\max(|I[i]^{k+1} - I[i]^k|) > \text{tolerance}$, then advance the iteration count and go to step 7.
- 10 Print the converged load flow solution and Stop

V. PROPOSED UNBALANCED THREE PHASE DISTRIBUTION LOAD FLOW METHOD

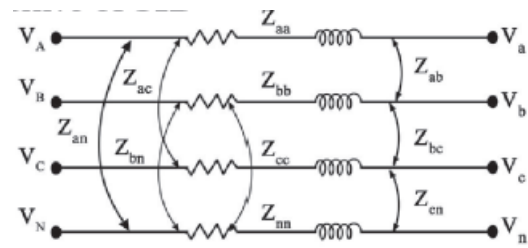


Fig. 3. Three Phase Line Section Model

Fig. 3 shows a three-phase line section between bus- ' i ' and bus- ' j '. The primitive impedance of this line can be solved using method developed by Carson and Lewis [13].

A 4×4 matrix which takes into account the self and mutual coupling effects of the unbalanced three phase line section is expressed as

$$[Z_{abcn}] = \begin{bmatrix} Z_{aa} & Z_{ab} & Z_{ac} & Z_{an} \\ Z_{ba} & Z_{bb} & Z_{bc} & Z_{bn} \\ Z_{ca} & Z_{cb} & Z_{cc} & Z_{cn} \\ Z_{na} & Z_{nb} & Z_{nc} & Z_{nn} \end{bmatrix} \quad (21)$$

After Kron's reduction technique [6] is applied, the above 4×4 matrix is then reduced to phase impedance 3×3 matrix which includes the effects of the neutral or ground wire as shown as

$$[Z_{abc}] = \begin{bmatrix} Z_{aa-n} & Z_{ab-n} & Z_{ac-n} \\ Z_{ba-n} & Z_{bb-n} & Z_{bc-n} \\ Z_{ca-n} & Z_{cb-n} & Z_{cc-n} \end{bmatrix} \quad (22)$$

The relation between the bus voltages and branch currents of the three phase line can be expressed as

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} V_A \\ V_B \\ V_C \end{bmatrix} - \begin{bmatrix} Z_{aa-n} & Z_{ab-n} & Z_{ac-n} \\ Z_{ba-n} & Z_{bb-n} & Z_{bc-n} \\ Z_{ca-n} & Z_{cb-n} & Z_{cc-n} \end{bmatrix} \begin{bmatrix} I_{Aa} \\ I_{Bb} \\ I_{Cc} \end{bmatrix}$$

In a three-phase line if any of the phases are not present, then its corresponding row and column elements in the matrix contains zeros.

The above proposed algorithm can be extended to a multi-phase line section or bus easily. For example, if a three-phase line connected between bus- ' i ' and bus- ' j ', then its corresponding branch B_i of $[B]$ matrix will be a 3×1 vector, +1 element in the BIBC matrix is replaced by a 3×3 identity sub matrix and z_{ij} element in the BCBV matrix is replaced by 3×3 primitive line impedance matrix. Similarly the elements at each bus of DLF matrix are 3×3 sub matrices

VI. TEST RESULTS

The proposed radial distribution load flow algorithm is implemented using Matlab software with the convergence tolerance was at 0.0001 p. u.

A. Test Results of single phase Radial Distribution Systems

The proposed distribution load flow method was tested on IEEE 15 bus, IEEE 34bus,IEEE 69 bus and IEEE 85 bus radial distribution systems and the load flow results obtained. Table 1 shows the active power and reactive power losses occurred in the various test systems.

Table I

S. No.	Test System	Active Power Loss (KW)	Reactive Power Loss (KW)
1	15 Bus	61.78	57.28
2	34 Bus	221.67	65.09
3	69bus	224.97	102.15
4	85bus	315.60	198.28

There was a compiling problem while using the method in [10] to obtain the load flow solution for 69 bus, 85 bus indicating the large size of two dimensional array declarations of BIBC, BCBV and DLF matrices which contain complex elements.

But this problem is not present in the proposed method for the same above test systems. The number of iterations required for convergence in the proposed algorithm is reduced when compared with branch current load flow method [12] as shown in table 2. The memory requirement and no error in compilation will be main advantages in using this load flow algorithm even for large distribution systems.

Table II

S. No.	Test System	No of Iterations for Tolerance of 0.0001 p.u.	
		Branch Current Load Flow Method [12]	Proposed Algorithm
1	15 Bus	7	3
2	34 Bus	7	3
3	69 Bus	8	4
4	85 Bus	10	4

B. Test Results of Three phase Radial Distribution Systems

Fig.4 shows the practical unbalanced three phase distribution feeder emanating from Pathardhi 132/11KVGrid substation in India [15].

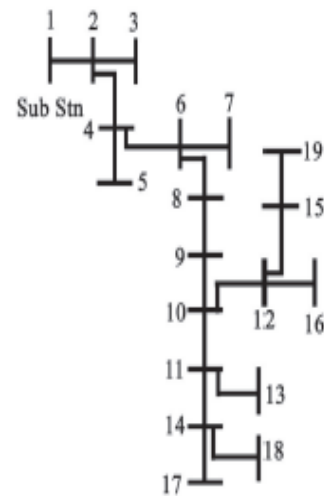


Fig. 4. A Practical Distribution Feeder

The three phase load flow solution has taken only three iterations for convergence with a tolerance value of 0.0001 p.u. The voltage values of load flow results are tabulated in table 3.

Table III

S. No.	Phase A Voltage (p.u.)	Phase B Voltage (p.u.)	Phase C Voltage (p.u.)
1	1.0000	1.0000	1.0000
2	0.9860	0.9878	0.9866
3	0.9848	0.9876	0.9857
4	0.9803	0.9820	0.9810
5	0.9801	0.9819	0.9809
6	0.9768	0.9786	0.9777
7	0.9765	0.9783	0.9775
8	0.9696	0.9708	0.9704
9	0.9619	0.9620	0.9617
10	0.9510	0.9503	0.9497
11	0.9496	0.9490	0.9478
12	0.9494	0.9484	0.9482
13	0.9493	0.9485	0.9472
14	0.9490	0.9485	0.9473
15	0.9470	0.9456	0.9456
16	0.9486	0.9471	0.9474
17	0.9481	0.9480	0.9467
18	0.9487	0.9481	0.9469
19	0.9457	0.9440	0.9447

VII. CONCLUSIONS

Thus, a simple and efficient load flow algorithm is proposed which does not require any LU decomposition and formation of BIBC and BIBV matrices. There by one can obtain the load flow solution with faster convergence which makes it to applicable to the larger distribution systems also.

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