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Image Inpainting Using Second-Order Derivative-Based Autoregressive Model

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***Abstract:** This paper introduces a new method, based full range autoregressive model, for image inpainting. The model parameters are estimated using iterative technique and Bayesian methods. Based on the parameters, coefficients of the model are computed. Autocorrelation coefficients are computed using the model coefficients. Based on the autocorrelation, the damaged region or noised regions are identified. The identified region is inpainted the full range autoregressive model. The performance of the proposed method is measured based on the MSE and PSNR values. The obtained results reveal that the proposed method yields better results than the existing methods.*

***Keywords:** Inpainting; Autoregressive Model; Bayesian Method; Iterative Technique; Query Image; Target Image.*

I. INTRODUCTION

Image inpainting is one of the fundamental methods of image processing that plays a noteworthy role in modifying the image, i.e. recover or restore the information lost by using the surrounding information. Image inpainting is an image restoration problem in the form of image interpolation, where a marked area is filled by its neighborhood information. The inpainting is applied in many areas like reconstruction of damaged or missing data in wireless transmission to removal of cracks; removal or recover the blocking effects in video frames which broadcasted in television; scratches, noise, dust or strain marks of scanning glass on the scanned image, stamped date and red-eye from photographs; and the infamous airbrushing of political enemies, etc. Thus, it plays a significant role in day-to-day activities. At the earlier stage of the restoration and inpainting, many researchers used partial differential equation model [1,2]. In recent years, most of the approaches have been developed based on partial differential equations and calculus of variations to structured or non-textured images. The mathematical and statistical models like partial differential equation [1,2], autoregressive models [4,5,6], markov random field models [7,8], time series models [9,10,11], play an important role in inpainting and restoration. Generally, the inpainting problem could be expressed as an image with a masked region, fill-in each pixel inside with the neigbouring pixel values. This practical importance of restoring and modifying images, ultimately, results in image inpainting. The partial differential equation based inpainting algorithms lies upon diffusion and variation in formulations, which have been successfully employed for piecewise smooth images while the gap between the damaged and neighboring regions is thinner than the surrounding objects. The autoregressive model predicts the linear relationship among the pixels in a region or the whole image as well as the markov random field model describes texture properties in well and good manner.

The parameter estimate plays a significant role in precision of restoration and inpainting. Despite, the artificial neural network based method consumes time than the others like least square (LE), maximum likelihood method (MLE), Bayesian method, iterative method, it gives precise estimate of results. Since the model proposed in [6] has the properties of the autoregressive model and the markov random field model, this paper exploits merits and avoids demerits of both model. The FRAR model is the most appropriate for image inpainting because it utilizes the linear dependency of the pixels in an image. The pixels in a region or an image linearly depend on each other.

II. PROPOSED INPAINTING METHOD

Let $X(k, l)$ be a random variable, which represents the intensity value of a pixel at location (k, l) in an image. Let the intensity values, X, include noise which is independent and identically distributed to a Gaussian random process. The noise is denoted by $\varepsilon(k, l)$, i.e. $\varepsilon(k,l) \sim N(0,\sigma^2)$. Thus, the images are assumed to be a Gaussian random process.

Since $\{X(s); s \in S\}$ is a stochastic process, where $S=\{s: (k,l); 1 \le k, l \le M\}$, $\{X(s)\}$ can be considered as a Markov process as it has the conditional probability,

$$P\{X(s_n) = i_n | X(s_k) = i_k : k = 0,1,2,\ldots,n-1\} = P\{X(s_n) = i_k | X(s_{n-1}) = i_{n-1}\}$$

for all $i_k, k = 0,1,2,\ldots,n-1$ and s_k belonging to the state space S and $s_0 < s_1 < \ldots < s_n$.

Thus, we propose an FRAR model as in equation (1),

$$X(k,l) = \sum_{p=-M}^{M} \sum_{\substack{q=-M \\ p=q\neq 0}}^{M} \Gamma_r X(k+p,\ l+q) + \varepsilon(k,l) \tag{1}$$

where, $$\Gamma_r = \frac{K \sin(r\theta)\cos(r\phi)}{\alpha^r} \tag{2}$$

and K, α, θ and ϕ are real parameters. The Γ_rs are the model coefficients, which are computed by substituting the model parameters K, α, θ and ϕ in equation (2). The model parameters are interrelated.

In fact, the pixel $X(k, l)$, in an image, has regression on its neighborhood pixels. However, in this case, the dependence of $X(k, l)$ on neighborhood values may be true to some extent. In fact, the process is Gaussian under the assumption that the $\varepsilon(k, l)$s are Gaussian, and in this case, it establishes a probabilistic structure. The range of the parameters of the model is set with the constraints $K \in R$, $\alpha > 1$, $0 < \theta < \pi$, $0 < \phi < \pi/2$.

III. Parameter Estimation

To implement the proposed FRAR model, the parameters have to be estimated. The parameters, K, α, θ, and ϕ are estimated by taking suitable prior information for the hyper parameters β, γ, and δ, based on iterative Bayesian methodology. The hyper parameters mean the parameters of the prior distribution of the actual parameters K, α, θ, and ϕ of the model proposed in equation (1). Since, the error term $\varepsilon(k, l)$ in equation (1) is a random process, which is independent and identically distributed to Gaussian process, the joint probability density function of the stochastic process $\{X(t)\}$ is

$$P\left(X/\Theta\right) \propto \left(\sigma^2\right)^{-N/2} \exp\left[-\frac{1}{2\sigma^2}\sum_{t=1}^{N}\left\{X_t - K\sum_{r=1}^{\infty} S_r X_{t-r}\right\}^2\right] \quad (3)$$

where $X = (X_1, X_2, \ldots, X_N)$; $\Theta = (K, \alpha, \theta, \phi, \sigma^2)$ and

$$S_r = \frac{\sin(r\theta)\cos(r\phi)}{\alpha^r}.$$

The range of the index r, viz. 1 to N, while we analyse the real data with N observations, and so the joint probability density function of the observations is given in equation (3). The summation $\sum_{r=1}^{\infty}$ can be replaced by $\sum_{r=1}^{N}$, which gives

$$P\left(X/\Theta\right) \propto \left(\sigma^2\right)^{-N/2} \exp\left[-\frac{1}{2\sigma^2}\sum_{t=1}^{N}\left\{X_t - K\sum_{r=1}^{N} S_r X_{t-r}\right\}^2\right] \quad (4)$$

By expanding the square in the exponent, we get

$$P\left(X/\Theta\right) \propto \left(\sigma^2\right)^{-N/2} \exp\left[-\frac{1}{2\sigma^2}\left\{\begin{array}{l} T_{00} + K^2\sum_{r=1}^{N} S_r^2 T_{rr} \\ +2K^2 \sum_{\substack{r,s=1\\ r<s}}^{N} S_r S_s T_{rs} - 2K\sum_{r=1}^{N} S_r T_{0r}\end{array}\right\}\right] \quad (5)$$

where

$$T_{rs} = \sum_{t=1}^{N} X_{t-r} X_{t-s}, \qquad r, s = 0, 1, 2, \ldots, N$$

The above joint probability density function can be written as

$$P\left(X/\Theta\right) \propto \left(\sigma^2\right)^{-N/2} \exp\left[-\frac{Q}{2\sigma^2}\right] \quad (6)$$

where

$$Q = T_{00} + K^2\sum_{t=1}^{N} S_r^2 T_{rr} + 2K^2\sum_{\substack{r,s=1\\ r<s}}^{N} S_r S_s T_{rs} - 2K\sum_{r=1}^{N} S_r T_{0r}$$

$K \in R$, $\alpha > 1$, $0 < \theta < \pi$, $0 < \phi < \pi/2$ and $\sigma^2 > 0$.

The joint prior density function of Θ is given by

$$P(\Theta) \propto \beta\exp(-\beta(\alpha-1) - \nu/\sigma^2)(\sigma^2)^{-(\delta+1)}; \quad \sigma^2 > 0,\ \alpha > 1,\ 0<\theta<\pi,\ 0<\phi<\pi/2. \quad (7)$$

where, P is a general notation for the probability density function of the random variables given within the parentheses following P.

Using (6), (7), and Bayes theorem, the joint posterior density of K, α, θ, ϕ, and σ^2 is obtained as

$$P\left(\Theta/X\right) \propto \exp(-\beta(\alpha-1))\exp(-1/2\sigma^2)(Q+2\nu)\left(\sigma^2\right)^{-\left(\frac{N}{2}+\delta+1\right)}; \quad (8)$$

$K \in R$, $\alpha > 1$, $0 < \theta < \pi$, $0 < \phi < \pi/2$ and $\sigma^2 > 0$.

Integrating (8) with respect to σ^2, the posterior density of K, α, θ, and ϕ is obtained as

$$P(K, \alpha, \theta, \phi / X) \propto \exp(-\beta(\alpha-1))(Q+2\nu)^{-\left(\frac{N}{2}+\delta\right)}; \quad (9)$$
$$K \in R,\ \alpha > 1,\ 0<\theta<\pi,\ 0<\phi<\pi/2$$

where,

$$[Q+2\nu] = \left[\left(K^2\sum_{r=1}^{N} S_r^2 T_{rr} + 2K^2\sum_{\substack{r,s=1\\ r<s}}^{N} S_r S_s T_{rs} - 2K\sum_{r=1}^{N} S_r T_{0r}\right) + T_{00} + 2\nu\right] \quad (10)$$

That is,

$$\begin{aligned}(Q+2\nu) &= aK^2 - 2Kb + T_{00} + 2\nu, \\ &= C\left[1 + a_1\left(K - b_1\right)^2\right]\end{aligned} \quad (11)$$

where,

$$C = T_{00} - \frac{b^2}{a} + 2\nu$$

$$a = \sum_{r=1}^{N} S_r^2 T_{rr} + 2\sum_{\substack{r,s=1\\ r<s}}^{N} S_r S_s T_{rs}$$

$$b = \sum_{r=1}^{N} S_r T_{0r} \; ; \; a_1 = \frac{a}{C} \; ; \; \mathbf{b_1} = \frac{\mathbf{b}}{\mathbf{a}}$$

Thus, the above joint posterior density function of K, α, θ, and ϕ can be rewritten as

$$P(K, \alpha, \theta, \varphi / X) \propto \exp(-\beta(\alpha-1))\left[C\left\{1+a_1(K-b_1)^2\right\}\right]^{-d} \quad (12)$$

$$K \in R,\ \alpha > 1, 0 < \theta < \pi,\ 0 < \varphi < \pi/2$$

where, $d = \frac{N}{2} + \delta$

This shows that given α, θ, and ϕ the conditional distribution of K is 't' distribution located at b_1 with (2d–1) degrees of freedom.

The proper Bayesian inference on K, α, θ, and ϕ can be obtained from their respective posterior densities. The joint posterior density of α, θ, and ϕ, namely, P(α, θ, ϕ / *X*), can be obtained by integrating (12) with respect to K. Thus, the joint posterior density of α, θ, and ϕ is obtained as

$$P(\alpha, \theta, \varphi / X) \propto \exp(-\beta(\alpha-1)) C^{-d} a_1^{-1/2} \quad (13)$$

$$\alpha > 1,\ 0 < \theta < \pi,\ 0 < \phi < \pi/2$$

The marginal posterior density of α, θ, and ϕ in (13) is a complicated function and is analytically not solvable. Therefore, we can find the original posterior density of α, θ, and ϕ numerically from the joint density (13).

That is,

$$P(\alpha) \propto \iint P(\alpha, \theta, \phi / X)\, d\theta\, d\phi$$

Similarly,

$$P(\theta) \propto \iint P(\alpha, \theta, \phi / X)\, d\alpha\, d\phi, \text{ and} \quad (14)$$

$$P(\phi) \propto \iint P(\alpha, \theta, \phi / X)\, d\alpha\, d\theta$$

The point estimates of the parameters α, θ, and ϕ may be taken as the means of the respective marginal posterior distribution, i.e. posterior means. With a view to minimize the computations, we first obtain the posterior mean of α numerically. Then fix α at its posterior mean and evaluate the conditional means of θ and ϕ fixing α at its mean. One can fix α, θ, and ϕ at their posterior means and evaluate the conditional mean of K.

Thus, the estimates are

$$\begin{aligned} \hat{\alpha} &= E(\alpha) \\ (\hat{\theta}, \hat{\phi}) &= E(\theta, \phi / \alpha = \hat{\alpha}) \quad \text{and} \\ \hat{K} &= E(K / \hat{\alpha}, \theta = \hat{\theta}, \phi = \hat{\phi}). \end{aligned} \quad (15)$$

The estimated parameters K, α, θ, and ϕ are adopted to compute the coefficients Γ_rs of the model in equation (1); and they were applied to restore damaged images. The damaged image could be missing of a particular region or heavily affected by noises.

The autocorrelation function (ρ_k) is derived from the model coefficients Γ_rs as follows:

$$\rho_1 = \frac{\Gamma_1}{1-\Gamma_2}$$

$$\rho_2 = \frac{\Gamma_1^2 + \Gamma_1 - \Gamma_2^2}{1-\Gamma_2}$$

Similarly, the kth order autocorrelations can be obtained by solving the equation (16) using recurrence relation. Their pattern is governed by the second order linear difference equation.

$$\rho_k = \Gamma_1 \rho_{k-1} + \Gamma_2 \rho_{k-2} \; ; \quad (16)$$

$$1 \le k \le m$$

From equation (16), the autocorrelation coefficients (ρ_k) are computed. To identify the noised region or the region damaged, one can test the significance of the autocorrelation. The test statistic [Pena02] for autocorrelation is defined as follows.

$$D_m = n\left[1 - \left|\tilde{R}_m\right|^{1/m}\right]$$

where, $\tilde{R}_m$ is the correlation matrix built by using the standardized autocorrelation coefficients $\tilde{\rho}_k$. That is,

$$\tilde{R}_m = \begin{bmatrix} 1 & \tilde{\rho}_1 & \cdots & \tilde{\rho}_m \\ \tilde{\rho}_1 & 1 & \cdots & \tilde{\rho}_{m-1} \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{\rho}_m & \tilde{\rho}_{m-1} & \cdots & 1 \end{bmatrix}$$

where, $\tilde{\rho}_k^2 = \frac{(n+2)}{(n-k)} \rho_k^2$, *n* is the number of samples and *m* is the lag variable.

The significance test on the statistic D_m could be carried out based on the measure $\pm \alpha(\sigma/\sqrt{n})$, where α is level of significance and σ is standard deviation. To make a confidence test at significance level α of the null hypothesis of no autocorrelation at lag *k*, one has to compare the value of sample coefficient with the

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aforementioned measure. If the sample coefficient falls outside the bands then the hypothesis is rejected at the level of significance α, i.e. the autocorrelation does exist among the data. Otherwise the null hypothesis is accepted. If the autocorrelation is highly significant, then the region is identified as the responses towards the region is not damaged or not affected by noise. Otherwise, the region is identified that the responses towards damaged region or affected by noise.

IV. EXPERIMENTS AND RESULTS

In order to examine the proposed method, first, the noise affected images were considered for the experiment; then information lost or damaged region of an image were subjected to the experiment. The proposed Full Range Autoregressive (FRAR) model is experimented with different kinds of images of size M×L. More than 550 textured and structured (natural) images were considered for the experiment. For a sample, a few textured images and structured images have been presented here. The textured images have been considered from standard Brodatz Album [12]. If the input image is color, it was converted to RGB color model; otherwise, it considered as it is for the experiment. The noise removal process was performed individually on each color components of the image, i.e. red, green, and blue. The color component was divided into various sliding window of size 3×3. Autocorrelation was computed on sliding window of each color component. The computed autocorrelation was tested at 5% level of significance. If the test is significant, then it was identified that the window was not damaged or not affected by noise. If the test is not significant, then it was concluded that the window was damaged or affected by noise. If the window was damaged or noised, the proposed FRAR model discussed in section was employed to restore the damaged or noised window. The experiment was conducted with different kinds of images. For a sample, due to space limitation, a few of them were presented in Figs. 1 and 2.

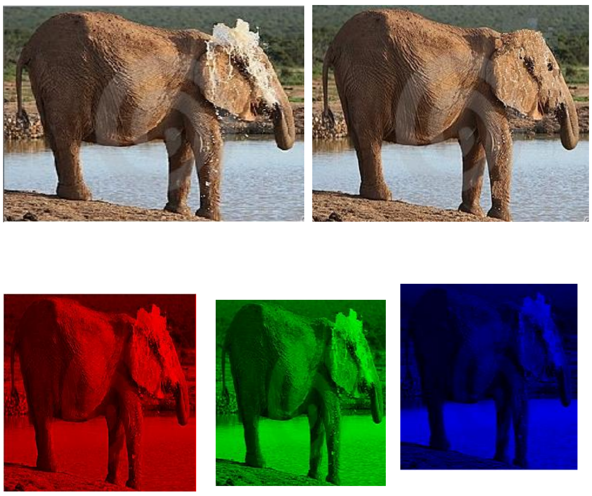

Fig. 1. (a): Actual Input Image; (b): Restored Image; (c): Red Component of the Image in (a); dD): Green Component of the Image in (a); (e): Blue Component of the Image in (a).

(a) (b)

Fig. 2. (a) Noise Affected Image; (b) Denoised Image.

In order to validate the proposed method, the PSNR was computed using the expression presented in equation (17). The PSNR values were tabulated in Table I.

$$PSNR = 10\log_{10}\left(\frac{(255)^2}{MSE}\right) \quad (17)$$

$$MSE = \frac{1}{M\times M}\sum_{i=0}^{M-1}\sum_{j=0}^{M-1}\left[f(i,j) - \hat{f}(i,j)\right]^2 \quad (18)$$

where f and $\hat{f}$ are the original and inpainted images of size M× L respectively.

Table II. PSNR and MSE Values for Different Types of Images at 5% Level of Significance

Images	**MSE**	**PSNR**
Girl	3.95	42.16
Elephant	3.21	43.15
Brick Wall	3.89	42.23
Sand	2.26	44.59

The obtained results reveal that the proposed method yields better results than the existing methods.

V. CONCLUSION

The FRAR model-based method was implemented for inpainting the damaged or noised regions in an image. The model parameters were estimated using the iterative technique with Bayesian method. Based on the estimated parameters, autocorrelation coefficient was computed, and tested at 5% level of significance. If the test is significant, then the region of interest left, i.e. the region was not damaged or noised. If the test is not significant, then the region of interest is inpainted based on the FRAR model. The Peak Signal to Noise Ratio (PSNR) was computed to evaluate the performance of the proposed method. The obtained output shows that the proposed method outperforms the existing methods.

VI. REFERENCES

[1] Peterl, P., Homann, S., Nedwed, F., Hoeltgen, L., and Weickert, J., From Optimised Inpainting with Linear PDEs towards Competitive Image Compression Codecs, Lecture Notes in Computer Science, 9431, 2016, pp. 63-74.

[2] Li, S.J. and Yao, Z.A., Image inpainting algorithm based on partial differential equation technique, The Imaging Science Journal, vol. 61, pp. 292-300.

[3] Bugeau, A., Bertalmio, M., Combining Texture Synthesis and Diusion for Image Inpainting. VISAPP 2009 - Proceedings of the Fourth International Conference on Computer Vision Theory and Applications, 2009, Portugal. 26-33.

[4] Joyeux, L., Boukir, S., Besserer, B. and Buisson, O., Reconstruction of degraded image sequences. Application to film restoration, Image Vis. Comput., (19), 2001, 503-516.

[5] Yang, J., Ye, X., Li, K., Hou, C., and Wang, Y., Color-Guided Depth Recovery From RGB-D Data Using an Adaptive Autoregressive Model, IEEE Transactions on Image Processing, 23(8), 2014, 3443-3457.

[6] K. Seetharaman,A Block-oriented Restoration in Gray-scale Images Using Full Range Autoregressive Model, Pattern Recognition, Vol. 45(4), 2012, pp. 1591-1601.

[7] Ruzic, Tijana, and Aleksandra Pizurica. “Context-aware Patch-based Image Inpainting Using Markov Random Field Modeling.” IEEE Transactions on Image Processing 24(1), 2015, 444–456.

[8] Batool, N., Chellappa, R. Detection and Inpainting of Facial Wrinkles Using Texture Orientation Fields and Markov Random Field Modeling. IEEE Transactions on Image Processing, 23(9), 2015, 3773 - 3788.

[9] Hooper, A.; Bekaert, D.; Spaans, K.; Arıkan, M. Recent advances in SAR interferometry time series analysis for measuring crustal deformation. Tectonophysics 2012, 514–517, 1–13.

[10] Hetland, E.A., Musé, P., Simons, M., Lin, Y.N., Agram, P.S., DiCaprio, C.J., Multiscale In SAR time series (MInTS) analysis of surface deformation, J. Geophys. Res.: Solid Earth, 117, 2012, 1-17.

[11] Henderson, S.T., Pritchard, M.E., Decadal volcanic deformation in the Central Andes Volcanic Zone revealed by InSAR time series, Geochemistry, Geophysics, Geosystems, 14(5), 2013, 1358-1374.

[12] Brodatz, P., Textures: A photographic album for artisters and designerss, Dover, New York, 1966.